# Hypothesis Testing for Proportions

## Lecture 27
## Section 9.3

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# Outline

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# Introduction

- Any question about a population must first be stated in terms of a population parameter.
- We will work with only two parameters:
  - The population mean $\mu$.
  - The population proportion $p$.

# Introduction

There are only two basic questions that we ask:

- What is the value of the parameter? (Estimation)
- Does the evidence support or refute a claim about the value of the parameter? (Hypothesis testing)

# Example

If we want to learn about the effectiveness of a new drug...

- What parameter do we use?
- Do we estimate the parameter?
- Or do we test a hypothesis?

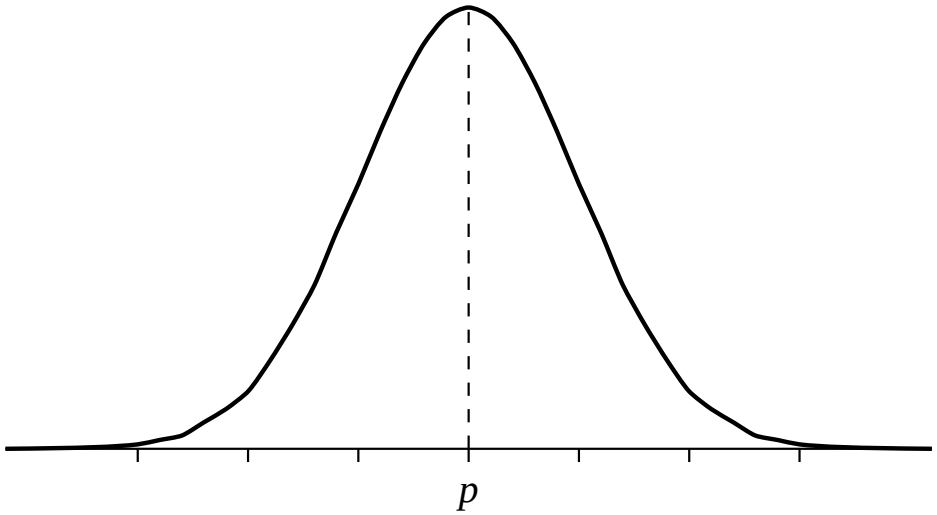
# Example

If we want to find out whether a newborn child is more likely to be male than female...

- What parameter do we use?
- Do we estimate the parameter?
- Or do we test a hypothesis?

# Outline

# The *p*-Value Approach



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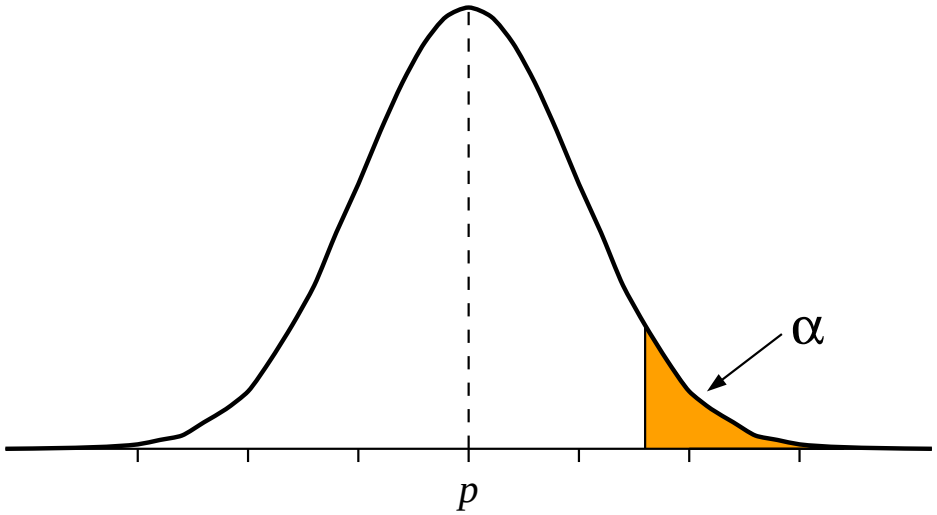

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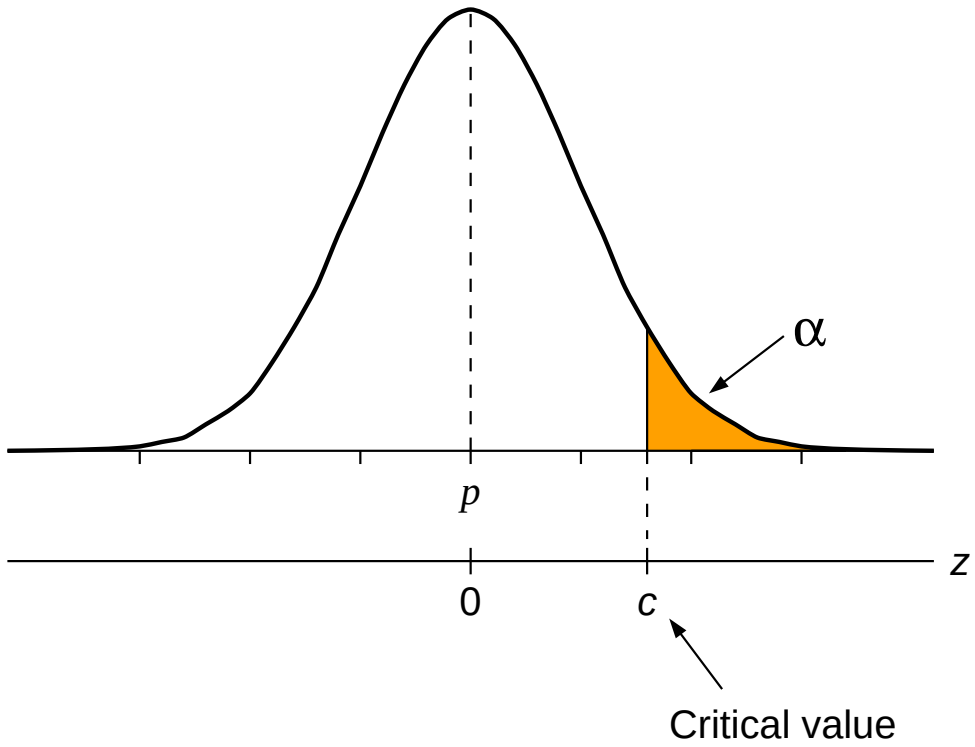

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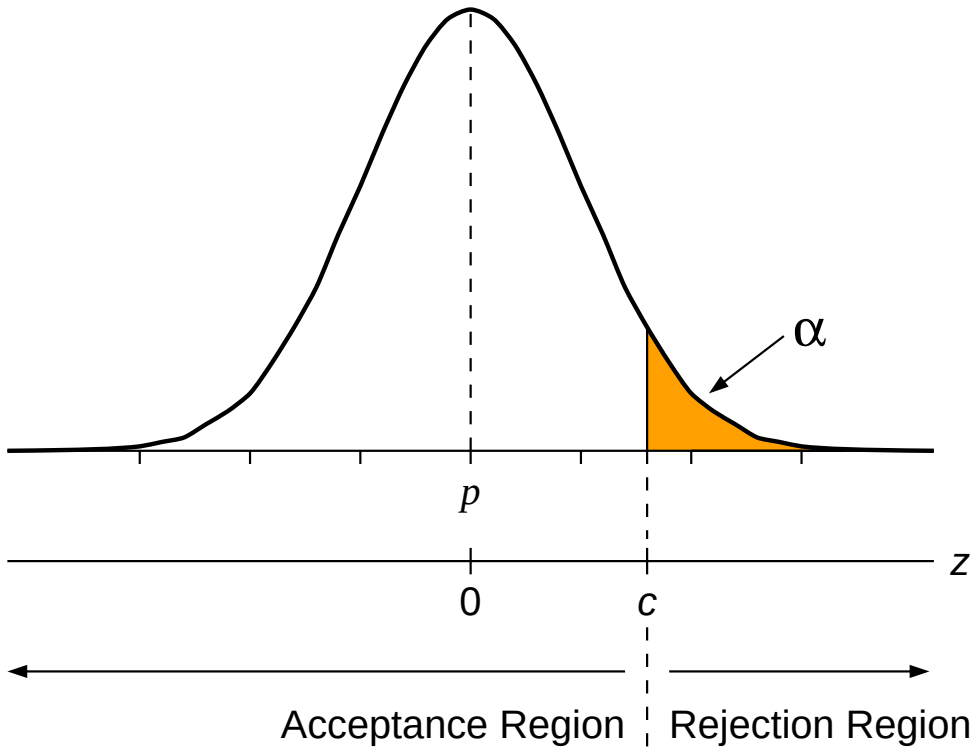

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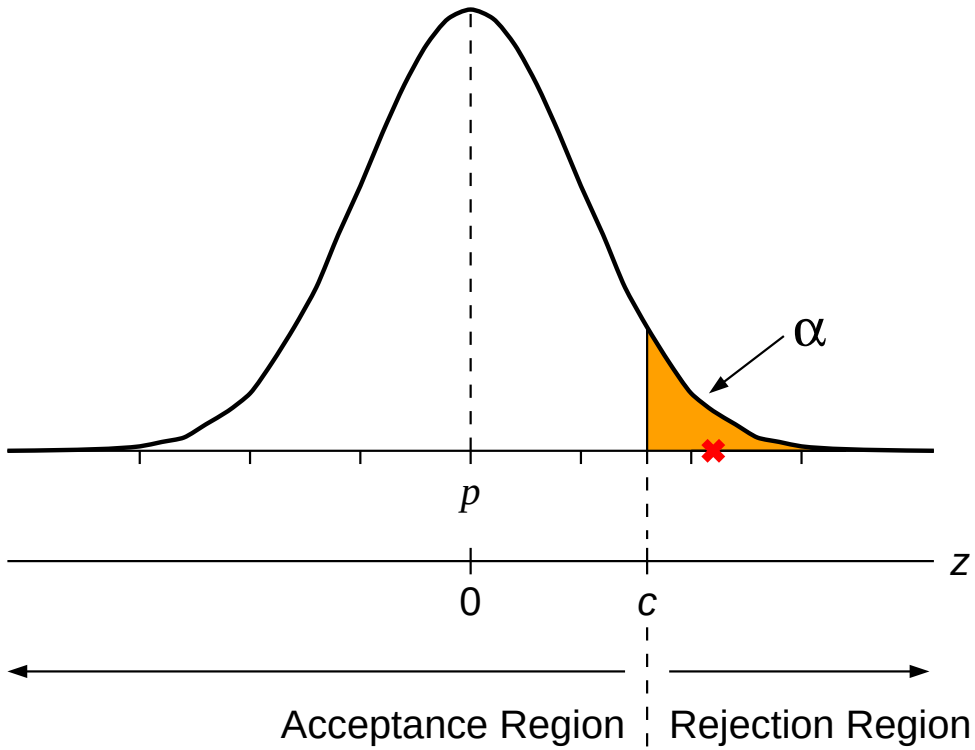

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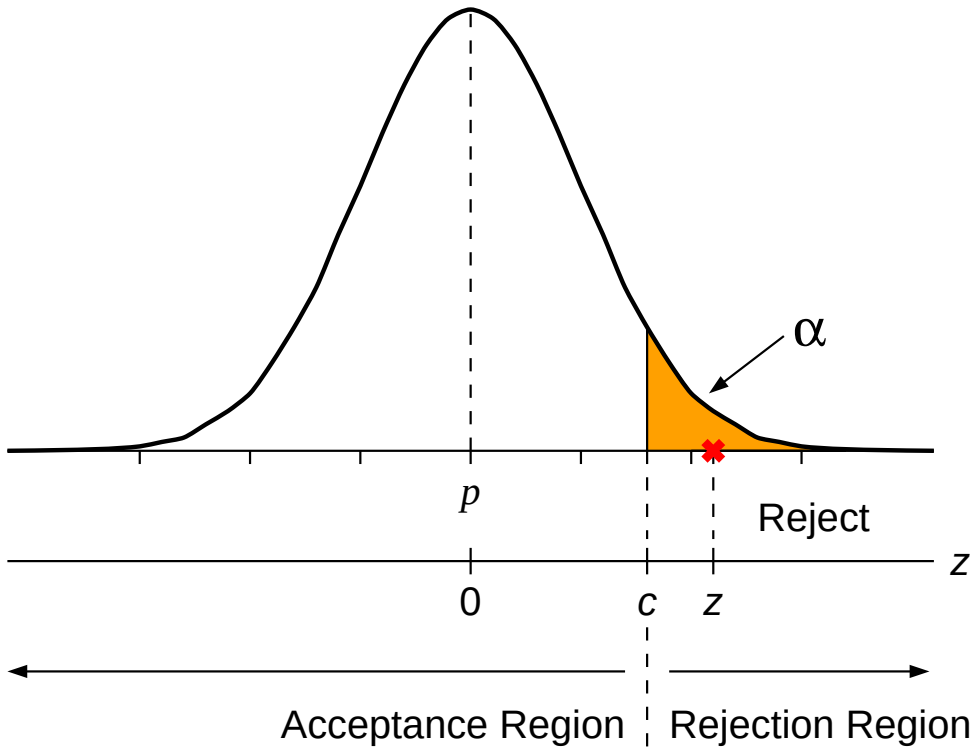

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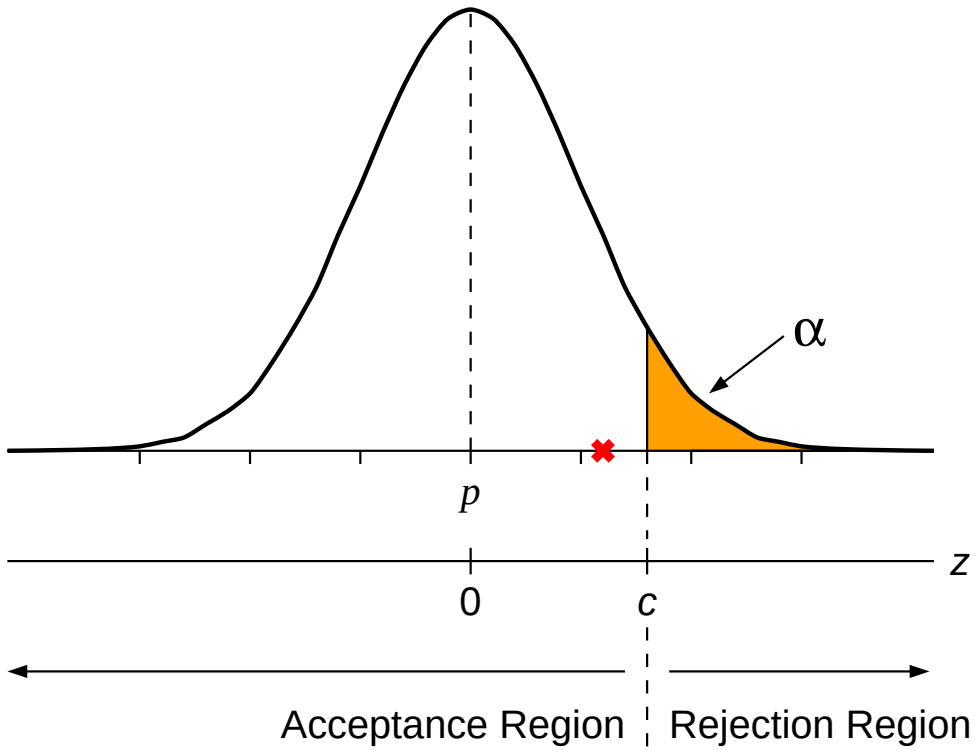

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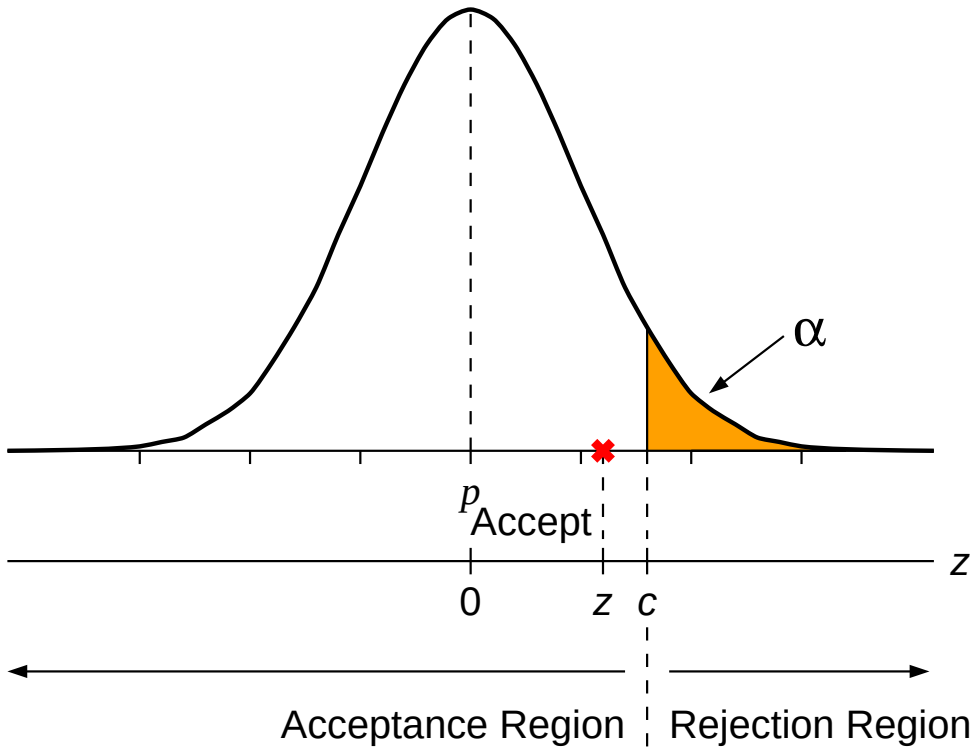

# Outline

# The Steps of Testing a Hypothesis

## *p*-Value Approach

The seven steps of hypothesis testing.

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5. Compute the *p*-value.

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5. Compute the *p*-value.
6. Make a decision.

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## *p*-Value Approach

The seven steps of hypothesis testing.

1. State the null and alternative hypotheses.
2. State the significance level.
3. State the formula for the test statistic.
4. Compute the value of the test statistic.
5. Compute the *p*-value.
6. Make a decision.
7. State the conclusion.

# Outline

# Example

## Example (Hypothesis testing)

- We want to test a coin for fairness?
- That is, does it land heads half the time?
- Suppose a random sample of 1000 coin tosses produces 525 heads and 475 tails.
- Test the hypotheses that coin is fair vs. not fair, at the 5% level of significance.

# Outline

# The Hypotheses

- Select the appropriate parameter ($p$) and describe what it represents.
- The null hypothesis should state a hypothetical value $p_0$ for the population proportion.

$$H_0 : p = p_0.$$

# The Hypotheses

- The alternative hypothesis must contradict the null hypothesis in one of three ways:
  - $H_1 : p < p_0$. (if direction of extreme is left.)
  - $H_1 : p > p_0$. (if direction of extreme is right.)
  - $H_1 : p \neq p_0$. (if direction of extreme is left and right.)

# The Hypotheses

## Example (Step 1)

(1) Let $p =$ proportion of tosses that land heads.
$H_0 : p = 0.50$.
$H_1 : p \neq 0.50$.

# Outline

# The Level of Significance

- Specify the level of significance $\alpha$.

# The Level of Significance

## Example (Step 2)

(2) $\alpha = 0.05$.

# Outline

# The Test Statistic

## Definition (Test statistic)

A test statistic is the statistic that is used to make the decision in a hypothesis test.

# The Test Statistic

**Definition (Test statistic)**

A test statistic is the statistic that is used to make the decision in a hypothesis test.

- What should the test statistic be in this situation?

# The Test Statistic

- According to the Central Limit Theorem, the statistic $\hat{p}$ has a normal distribution with

$$\mu_{\hat{p}} = p$$

and

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}.$$

# The Test Statistic

- Therefore, if the null hypothesis is true, then $\hat{p}$ is normal with mean $p_0$ and standard deviation $\sqrt{\frac{p_0(1-p_0)}{n}}$.

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$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}.$$

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$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}.$$

- The $z$-score has a standard normal distribution.

# The Test Statistic

## Example (Step 3)

(3) The test statistic is

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}.$$

# Outline

# The Value of the Test Statistic

- To compute the value of the test statistic, substitute the values obtained from the sample and from the null hypothesis.
- In this case, use the values of $\hat{p}$, $p_0$, and $n$.

# The Value of the Test Statistic

## Example (Step 4)

(4) From the null hypothesis, we have $p_0 = 0.50$.

$$p_0 = 0.50,$$
$$\hat{p} = \frac{525}{1000} = 0.525,$$
$$n = 1000,$$

Compute

$$\begin{aligned} Z &= \frac{0.525 - 0.50}{\sqrt{\frac{(0.50)(1-0.50)}{1000}}} \\ &= \frac{0.025}{0.01581} \\ &= 1.581. \end{aligned}$$

# Outline

# The *p*-Value

- To find the *p*-value, use the `normalcdf` function and the value of the test statistic ($z$).
- Pay attention to the direction of extreme as indicated by the alternative hypothesis.
  - To the left: *p*-value $=$ `normalcdf(-E99,`$z$`)`.
  - To the right: *p*-value $=$ `normalcdf(`$z$`,E99)`.
  - Two-sided: Find the area in the appropriate tail and *then double it*.

# The *p*-Value

## Example (Step 5)

(5)

$$\begin{aligned} p\text{-value} &= 2 \times \texttt{normalcdf(1.581,E99)} \\ &= 2 \times 0.0569 \\ &= 0.1138. \end{aligned}$$

# Outline

# The Decision

- The decision states whether to accept or reject the null hypothesis.
- If the *p*-value is less than $\alpha$, then write "Reject $H_0$."
- If the *p*-value is greater than $\alpha$, then write "Accept $H_0$."

# The Decision

## Example (Step 6)

(6) Accept $H_0$.

# Outline

# The Conclusion

- The conclusion restates the decision *in the language of the original problem*, without using any statistical jargon.
- It is enough to restate in plain English the hypothesis that was accepted.

# The Conclusion

## Example (Step 7)

(7) The proportion of heads is 50%.

(8) Or, the coin is fair.

# Summary

## Example (The seven steps)

(1) Let $p =$ proportion of tosses that land heads.
$H_0 : p = 0.50$.
$H_1 : p \neq 0.50$.

(2) $\alpha = 0.05$.

(3) The test statistic is $Z = \dfrac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$.

(4) $Z = \dfrac{0.525-0.50}{\sqrt{\frac{(0.50)(1-0.50)}{1000}}} = \dfrac{0.025}{0.01581} = 1.581$.

(5) $p$-value $= 2 \times$ `normalcdf(1.158,E99)` $= 0.1138$.

(6) Accept $H_0$.

(7) The proportion of heads is 50%.

# Outline

# Hypothesis Testing on the TI-83

## TI-83 Hypothesis Testing for $p$

- Press `STAT`.
- Select the `TESTS` menu.
- Select `1-PropZTest...`
- Press `ENTER`. A window appears.
- Enter the value of $p_0$.
- Press `ENTER` and the down arrow.
- Enter the numerator $x$ of $\hat{p}$.
- Press `ENTER` and the down arrow.
- Enter the sample size $n$.
- Press `ENTER` and the down arrow.

# Hypothesis Testing on the TI-83

## TI-83 Hypothesis Testing for $p$

- Select the type of alternative hypothesis.
- Press the down arrow.
- Select `Calculate`.
- Press `ENTER`.

# Hypothesis Testing on the TI-83

## TI-83 Hypothesis Testing for $p$

- The following appear in the display.
  - The title `1-PropZTest`.
  - The alternative hypothesis.
  - The value of the test statistic $Z$.
  - The $p$-value.
  - The value of $\hat{p}$.
  - The sample size $n$.

# TI-83 Example

## TI-83 *p*-value approach

- Work the example of 525 heads and 475 tails on the TI-83.

# Outline

# Assignment

## Homework

- Read Sections 9.1 - 9.2, pages 563 - 580.
- Let's Do It! 9.1, 9.2, 9.3, 9.4.
- Exercises 1 - 12, 14, page 580.